In math how is pi derived with high accuracy

Okay, let's dive into the fascinating world of how pi is derived with high accuracy! It's a story spanning millennia, and it's surprisingly complex. Here's a breakdown, progressing from ancient methods to modern techniques. I'll try to make it understandable without getting *too* bogged down in the technical details.

1. The Fundamental Concept: Defining Pi

Before diving into *calculating* pi, it's crucial to understand what it *is*. Pi (π) is defined as the ratio of a circle's circumference to its diameter.

* **Circumference (C):** The distance around the circle.
* **Diameter (d):** The distance across the circle through the center.
* **Pi (π) = C / d**

This ratio is *constant* for all circles, regardless of size. The challenge is to find that constant to as many digits as possible. It's an *irrational* number (it can't be expressed as a simple fraction) and *transcendental* (it's not the root of any polynomial equation with integer coefficients), meaning its decimal representation goes on forever without repeating.

2. Early Methods (Geometric Approaches - Before Calculus)

These methods relied on approximating the circle with polygons. The more sides the polygon has, the closer its perimeter gets to the circle's circumference.

*   **Archimedes (around 250 BC):** This is the foundation. Archimedes inscribed and circumscribed regular polygons (polygons with equal sides and angles) inside and outside a circle. He started with hexagons and doubled the number of sides repeatedly (12 sides, 24 sides, 48 sides, 96 sides...). He calculated the perimeters of these polygons.
    *   The perimeter of the inscribed polygon gives a *lower bound* for the circle's circumference.
    *   The perimeter of the circumscribed polygon gives an *upper bound*.
    *   He squeezed pi between these bounds, getting the approximation 3 10/71 < π < 3 1/7 (or roughly 3.1408 < π < 3.1429). This remarkably accurate estimate for the time was derived by hand.

*   **Later Geometric Refinements:** Mathematicians for centuries continued Archimedes' method, using more and more sides to improve accuracy. These methods were limited by the computational difficulty of calculating increasingly complex polygon perimeters by hand.

3. Infinite Series (Calculus Era - 17th Century and Beyond)

The development of calculus revolutionized pi calculation. Infinite series allowed for much more efficient approximations.

*   **Madhava of Sangamagrama (14th Century - India):** Madhava discovered several infinite series for pi, including:

    *   π = 3 + 4/(2*3*4) - 4/(4*5*6) + 4/(6*7*8) - 4/(8*9*10) + ... (This is a Leibniz series variant).
    *   He calculated pi to 11 digits using this method!

*   **Gottfried Wilhelm Leibniz (17th Century):** Rediscovered and popularized the series:
    *   π/4 = 1 - 1/3 + 1/5 - 1/7 + 1/9 - ... (Leibniz formula for π)
    *   This series is simple, but converges *very slowly* (meaning you need a huge number of terms to get good accuracy).

*   **Machin-like Formulas (John Machin, 1706):** Machin discovered a much faster converging formula based on the arctangent function:
    *   π/4 = 4 * arctan(1/5) - arctan(1/239)
    *   The arctangent function can be expanded into an infinite series (the Gregory-Leibniz series for arctan). The cleverness here is choosing values (1/5, 1/239) that make the series converge quickly. Mathematicians have found even better Machin-like formulas over the years.

* **Bailey–Borwein–Plouffe Formula (BBP, 1995):** A truly groundbreaking formula. The BBP formula allows you to calculate *any individual hexdecimal digit* of pi without needing to calculate the preceding digits! This is amazing because it doesn't require a massive computational effort for a specific digit.

4. Modern Methods (Computer Age - 20th and 21st Centuries)

With the advent of computers, pi calculation exploded.

*   **Gauss-Legendre Algorithm (1975, rediscovered):** An iterative algorithm that quadratically converges, meaning the number of correct digits roughly doubles with each iteration. This was used for many record-breaking calculations.
*   **Chudnovsky Algorithm (1987):** Based on Ramanujan's work with elliptic curves, this algorithm is even faster than Gauss-Legendre. It's the algorithm used for many of the current record-holding pi calculations. It gives around 14 digits of accuracy per iteration.
*   **Arithmetic-Geometric Mean (AGM) methods:** Based on the arithmetic-geometric mean, these are very efficient for large-scale pi calculations.
* **Distributed Computing:** Projects like the Great Internet Mersenne Prime Search (GIMPS) and others have used the combined power of thousands of computers to calculate trillions of digits of pi.

Where are we now? (as of 2024)

*   As of March 2024, the record for the most calculated digits of pi is over **100 trillion digits** (calculated by Emma Haruka Iwao using Google Cloud TPU).
*   Calculating pi is now mainly a benchmark for testing supercomputers and algorithms. It's not as much about finding new digits of pi itself, but rather pushing the limits of computational power and algorithm efficiency.

Key Takeaways:

*   **Pi's calculation has evolved from geometric approximations to sophisticated algorithms leveraging calculus and modern computing.**
*   **Infinite series (especially Machin-like formulas and Ramanujan-inspired formulas) have been crucial.**
*   **The algorithms used today rely heavily on number theory, analysis, and elliptic curves.**
*   **The main goal is no longer just to find digits of pi, but to test hardware, software, and algorithms.**

I hope this comprehensive explanation gives you a good understanding of how pi is derived with high accuracy! It's a fascinating journey that shows the power of mathematical ingenuity and the relentless pursuit of knowledge. Let me know if you have any further questions.